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Flow of Pastes in Bent Tubes

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Abstract

Processing requires that pastes flow in piping networks often into dies or moulds. Here paste flows in bent tubes of constant radius of curvature but varying angles have been investigated using a modified ram extruder. Measurements for two model pastes have shown that the pressure drop for bends below 105° is linearly dependent on the angle of bend, but at larger angles this relationship does not follow. Theoretical predictions from plasticity theory consistently underestimate pressure drops although the general form and magnitude is correct. The mechanisms controlling the flow, especially for larger angles, need to be investigated. © 1996 Elsevier Science Limited.

Notation

A	Minimum cross-sectional area of pipe for flow due to the stagnant region	(m^2)
c	Integration constant in Hencky's equation	(Mpa)
D	Dieland diameter	(m)
D_0	Piston diameter	(m)
k	Shear yield stress of bulk material under plastic deformation	(MPa)
L	Dieland length	(m)
p	Normal stress value at the centre of Mohr's circle	(MPa)
ΔP	'Extra' pressure drop due to stagnant region	(MPa)
P	Extrusion pressure	(MPa)
r	Radius of pipe	(m)
γ	Material velocity factor for lubricated shearing	(MPa s/m)
ϕ	Anticlockwise angle of rotation of α and β characteristic axes	(radians)
θ	Included half angle of stagnant zone	(radians)
σ_0	Bulk yield stress of material measured by extrusion	(MPa)
σ_x	Normal stress in x direction	(MPa)
σ_y	Normal stress in y direction	(MPa)
τ_0	Lubricated shear yield stress of material measured by extrusion	(MPa)
τ_{xy}	Shear stress in xy plane	(MPa)

Introduction

Paste extrusion is a popular and effective method of forming products as diverse as ceramics, catalysts and chocolate. Typical extrusion machinery includes ram extruders, screw and roll extruders and injection moulding equipment.

Since the flow of dry particles is a frictional process, a liquid phase is invariably added to form an extrusion paste. This liquid phase is commonly an aqueous solution of a rheology modifier such as clay or methyl cellulose although in the case of some ceramic injection moulding processes molten polymers are used. There is also a great range of lubricated materials, e.g. soaps, which can be included under the general heading of 'paste'. The flow of these extrusion pastes is extremely complex due to the relative movement of the liquid phase within the solid matrix. There is currently little understanding of the flow mechanics of these processes although a good semi-empirical expression is available due to Ovenston and Benbow.[1] These workers were the first to analyse the flow in a ram extruder as two distinct regions: a plastic deformation to account for the change of area due to extensional flow and a lubricated plug flow along the dieland of constant cross-section. They assumed that the flow in the dieland was lubricated by a layer of depleted solids concentration a few microns thick at the interface between the paste and the die surface. The resulting model predicts the extrusion pressure drop from a piston of diameter D_0 due to a bulk plastic deformation at the entry to the die of diameter D and a lubricated plug flow of velocity V in the dieland. The bulk yield stress of the material σ_0 characterises a pure plastic deformation that accounts for the change of area at the die entry. This is derived from a Tresca yield criterion which states that the material will undergo plastic yielding when the difference between the maximum and minimum principal stresses is equal to some critical value which is known as the tensile (or compressive) yield stress. This is added to the lubricated plug flow contribution at the die wall controlled by the lubricated shear yield stress τ_0 and the velocity factor γ.

$$P = 2\sigma_0 \ln\left[\frac{D_0}{D}\right] + 4\frac{L}{D}(\tau_0 + \gamma V) \qquad (1)$$

Ovenston and Benbow[1] present results for the flow of pastes in axisymmetric tubes which are well described by this model, although the flow of pastes in bent tubes has not been previously studied.

Plasticity theory of metals deformation provides an alternative method of analysis which is detailed by Hill.[2] Consider a planar stress field in the xy plane at a point within a plastic deformation envelope. Tresca's yield criterion defines the state of stress as

$$\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}{}^2 = k^2 \qquad (2)$$

where k is the yield stress of the material in pure shear, σ_x and σ_y are the normal stresses acting in the x and y directions respectively and τ_{xy} is the shear stress in the xy plane. If the derivatives of the stress tensor are continuous in both the x and y directions then taking the partial derivative of (2) with respect to x gives

$$\frac{1}{4}(\sigma_x - \sigma_y)\left[\frac{\partial \sigma_x}{\partial x} - \frac{\partial \sigma_y}{\partial x}\right] + \tau_{xy}\frac{\partial \tau_{xy}}{\partial x} = 0 \qquad (3)$$

Now if $\sigma_x = \sigma_y$ then $\frac{\partial \tau_{xy}}{\partial x} = 0$ and using the same argument in the y direction $\frac{\partial \tau_{xy}}{\partial y} = 0$. For stress equilibrium,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} \qquad (4)$$

hence

$$\frac{\partial \sigma_x}{\partial x} = \frac{\partial \sigma_y}{\partial y} = 0 \qquad (5)$$

if $\sigma_x = \sigma_y$, i.e. when the x and y axes are characteristics of maximum shear stress.

Mohr's circle at a point p gives us that

$$\sigma_x = -p - k \sin 2\phi \qquad (6)$$

$$\sigma_y = -p + k \sin 2\phi \qquad (7)$$

$$\tau_{xy} = k \cos 2\phi \qquad (8)$$

where ϕ is the anticlockwise rotation of the α characteristic with respect to the x axis.

Substituting from (6) and (7) into (5) then gives us

$$\frac{\partial p}{\partial x} + 2k\frac{\partial \phi}{\partial x} = \frac{\partial p}{\partial y} - 2k\frac{\partial \phi}{\partial y} = 0 \qquad (9)$$

which reduces to

$$dp + 2kd\phi = 0 \text{ along an alpha characteristic} \qquad (10)$$

$$dp - 2kd\phi = 0 \text{ along a beta characteristic} \qquad (11)$$

where eqns (10) and (11) are known as Hencky's equations. Hencky's equations can alternatively be presented in the integrated form relating a change in the centre point p of Mohr's circle of stress to a rotation ϕ of the characteristic stress axes α and β and the shear yield stress of the bulk material k.

$$\begin{aligned} p &= c + 2k\phi \text{ along an } \alpha \text{ line} \\ p &= c - 2k\phi \text{ along a } \beta \text{ line} \end{aligned} \qquad (12)$$

where c is an integration constant.

Experimental System

A laboratory ram extruder system under the control of Dartec strain frame allows for the measurement of extrusion pressure through dies fitted to the end of the piston (Fig. 1). A special die was constructed to allow extrusion through a stainless steel tube which could be attached and removed by means of compression fittings. Tubes were cut to identical 84 mm lengths and then bent to known angles at constant 15 mm radius of curvature by means of a hand-held tube bender. Pressure drops could hence be obtained for tubes of varying angle at various extrusion speeds. Tubes of internal diameter 4·5 mm, 5·92 mm and 7·76 mm were tested.

Two paste systems were used and these were formulated as shown in Table 1. Pastes are mixed dry for 20 min then the liquid phase added and wet mixed for a further 20 min. High shear pugging is then used to reduce the agglomeration and improve the homogeneity of the test pastes. The extrusion parameters (Table 2) of the test pastes were derived by the method of Benbow and Bridgwater.[3]

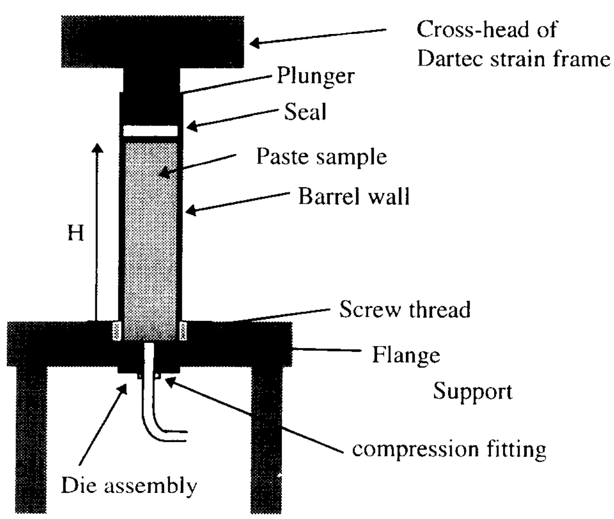

Fig. 1. Schematic diagram of experimental apparatus.

Table 1. Formulation of extrusion pastes

		Mix A *(g)*	*Mix B* *(g)*
Alumina	280 sieve	333	333
	400 sieve	333	333
	1500 sieve	333	333
Bentonite		38	42
Potato starch		38	—
Glucose		—	70
Water		190	140

Table 2. Extrusion parameters for test pastes after Benbow and Bridgwater[3]

Extrusion parameter	*Mix A*	*Mix B*
σ_0 (MPa)	0·35	0·10
τ_0 (MPa)	0·02	0·01
γ (MPa s/m)	0·40	0·42

Results and Discussion

Figure 2 shows the effects of the angle of bend on the pressure drop due to the bend for mix A in a 4·5 mm diameter tube with velocity as a parameter. The pressure drop due to the bend has been obtained by subtracting the pressure drop measured for an equivalent straight tube at the same velocity, found to be about 4·5 MPa for a straight 4·5 mm diameter tube at 1 mm/s extrudate velocity. The repeatability of these data is approximately 10% for angles below 105°, although for angles above this it reduces sharply to around 50%. The reason for this change remains unclear since the experiment is otherwise identical in all respects. A notable result is that the pressure drop appears to form a local minimum at around 135°. This conclusion is supported by results for mix B (Fig. 3) although in this case the data are more subject to error due to the lower overall pressure drops. Flow visualisation experiments are currently being attempted to determine the cause of this behaviour.

A significant result is that for both pastes A and B there appears to be little effect of changing the velocity of extrusion. If this is indeed the case and the pressure drop can be attributed to bulk plastic deformation, then Hencky's equations should describe the pressure drop due to the bend. Using a Tresca yield criterion the value of k can be taken as $0{\cdot}5\sigma_0$ which is 0·17 MPa for mix A and 0·05 MPa for mix B. Making the additional assumption that the characteristic axes are rotated by the same angle as the bend allows for prediction of the pressure drop due to the bend (Figs 2 and 3), the prediction due to Hencky's equations is a significant underestimate for mix A although the general shape would seem to be approximately correct (Fig. 2). A beam bending analogy would suggest a significant tension below the neutral axis which complicates the situation considerably since the tensile yield stress of a paste is invariably much less than its equivalent compressive yield value. It is therefore no surprise that this application of

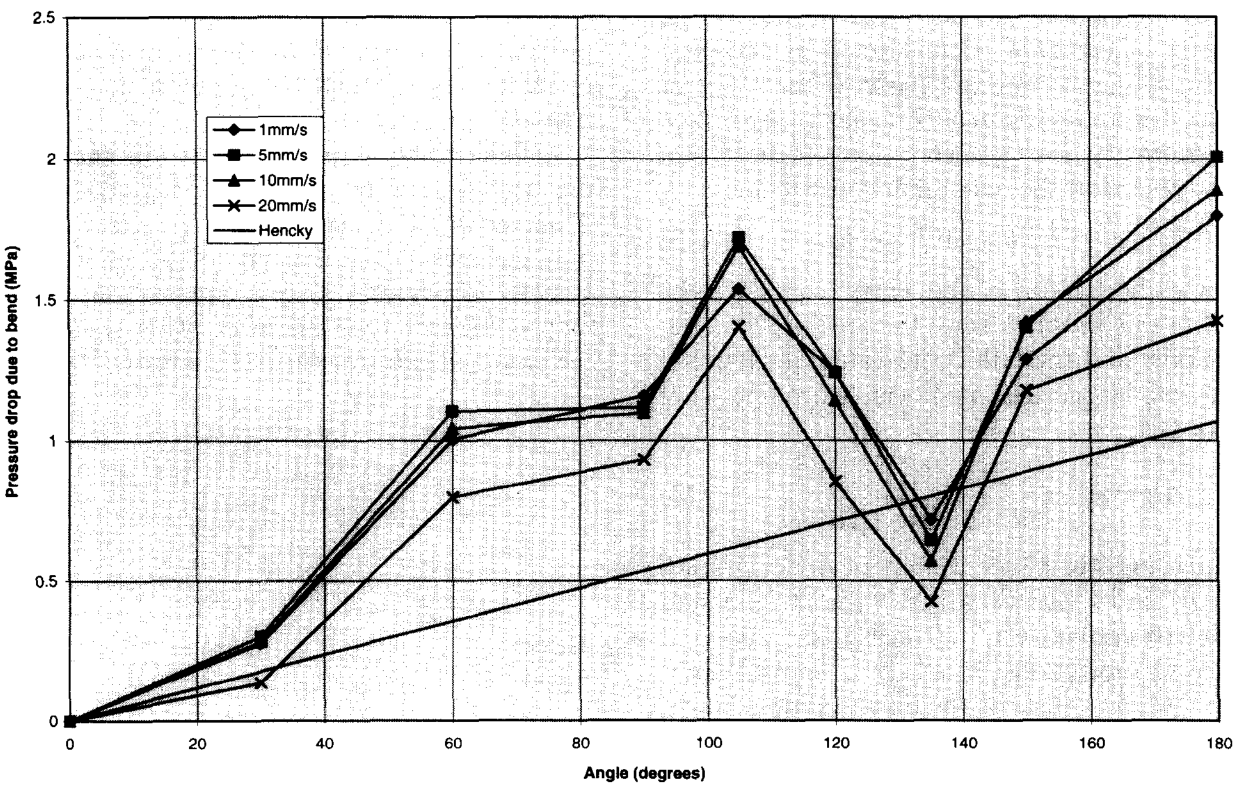

Fig. 2. Measured experimental pressure drops and Hencky predictions for paste A.

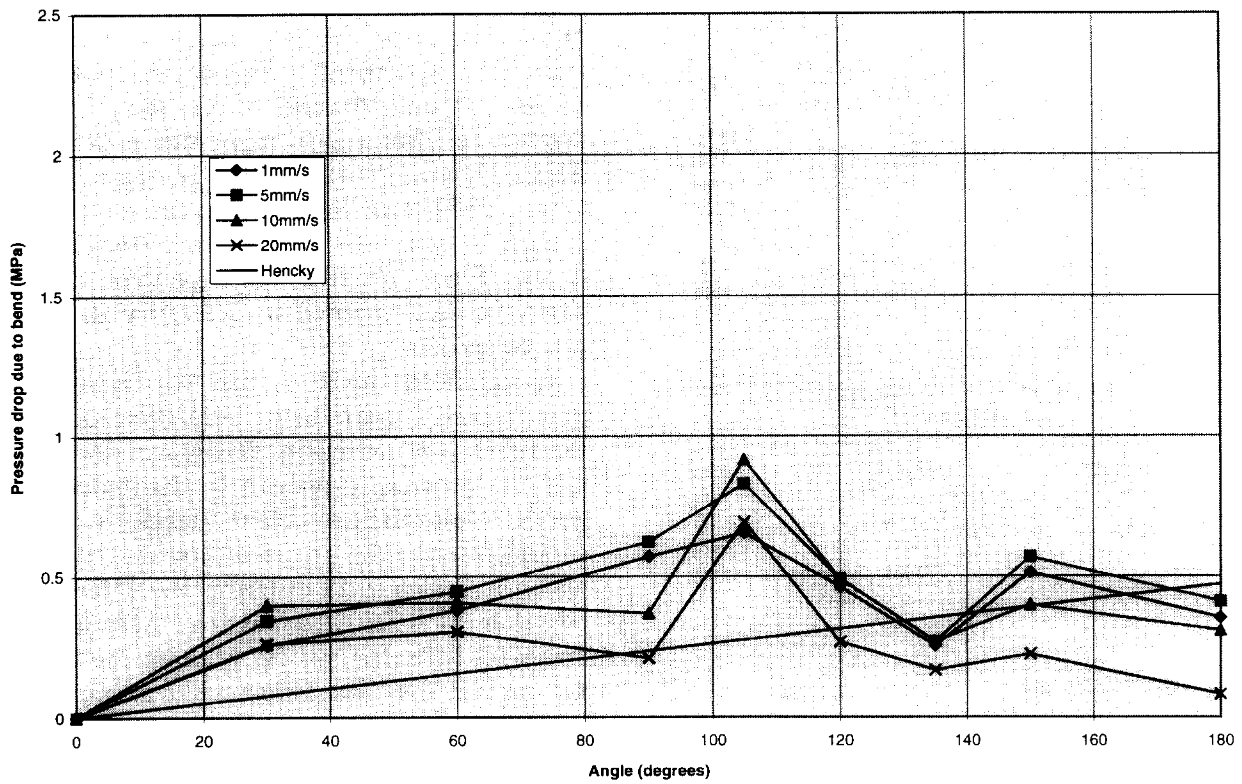

Fig. 3. Measured experimental pressure drops and Hencky predictions for paste B.

Hencky's equations produces only an approximation to the measured values. However, at low angles it would seem that a plastic deformation mechanism provides a plausible explanation of the measured pressure drops.

An alternative explanation for the deviations from Hencky's prediction can be found if the flow in the region of the bend is constricted by the presence of a stagnant region of paste. If the shape of this region is approximated as planar (Fig. 4) then simple geometry gives an expression for the size of the flow area A at the maximum constriction.

$$A = 2r^2(\theta - \sin\theta\cos\theta) \qquad (13)$$

Benbow's model for a die entry can then be applied to this problem since the area of the unrestricted pipe and the bulk yield value of the paste are known. Hence the predicted 'extra' pressure drop ΔP is

$$\Delta P = \sigma_0 \ln\left[\frac{\pi}{\pi - \theta + \sin\theta\cos\theta}\right] \qquad (14)$$

This equation is best solved numerically or graphically (Fig. 5) to give values of θ for the pressure drop differences between the. measured values and the Hencky predictions. It is then a simple matter to calculate the values of the dimensionless height h/r of the static zones (Fig. 6). Figure 7 shows the calculated profiles of the static zones required to account for the differences between the Hencky predictions and the experimentally determined pressure drops for pipes of differing angles. It would seem that there could be a stable stagnant zone for angles up to a critical point where the pressure drop becomes too great to sustain and the stagnant region is 'flushed' out. The exact mechanics are unclear at present, but it is thus prudent to establish in further work that the basis of the theory is sound by means of flow visualisation experiments.

A final point is that the pressure drop due to the bend would appear to become smaller as the diameter is increased to such an extent that it is only measurable for the tube of 4·5 mm diameter. Hencky provides a stress correlation which is independent of tube diameter. This is clearly not the case and calls for further analysis.

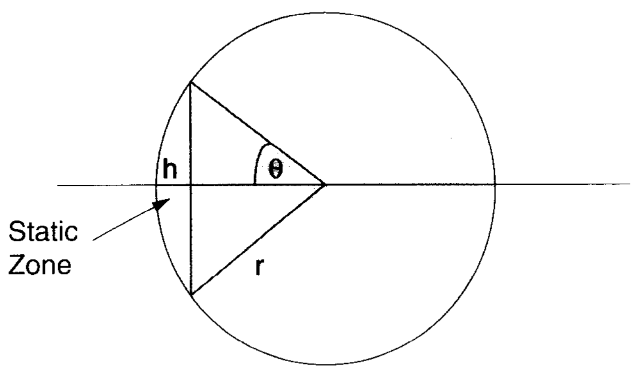

Fig. 4. Geometry of planar static zone.

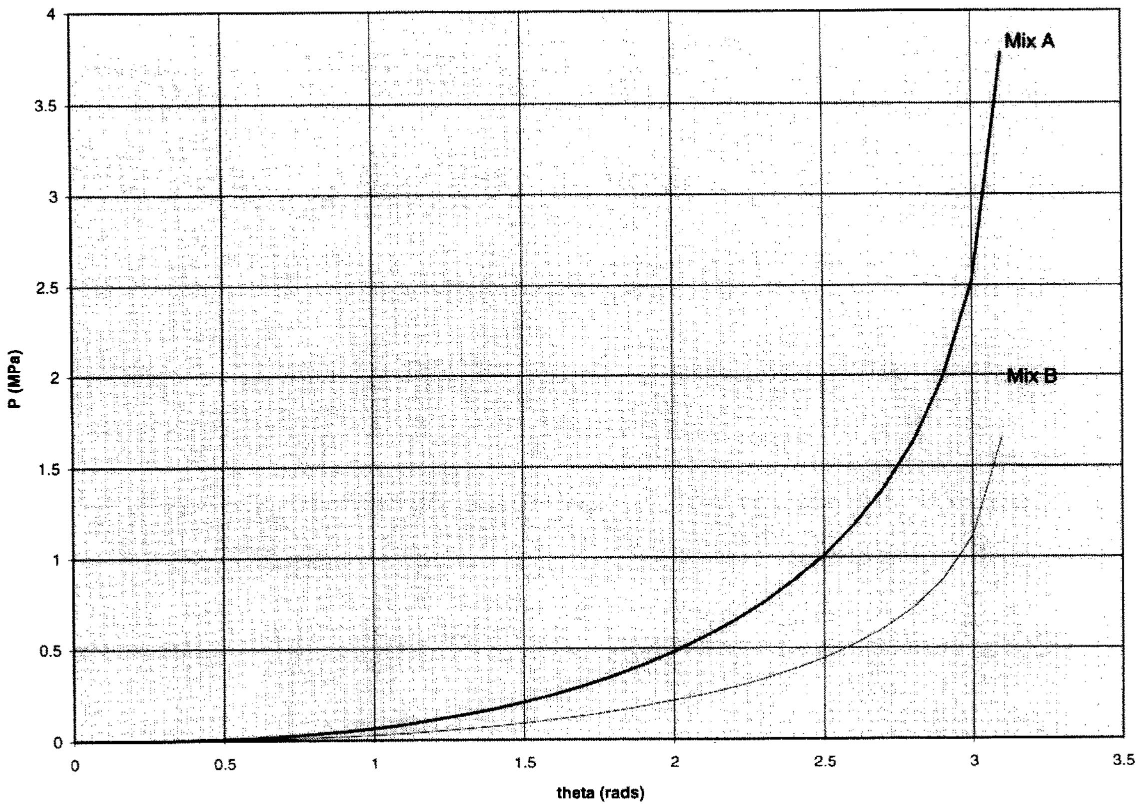

Fig. 5. Pressure drop P versus included half angle θ for the hypothesis of a planar static zone.

Conclusions

The flow of pastes around bends presents a complex theoretical problem although for angles less than 105° pressure drop predictions can be made by using Hencky's equations with an empirically determined k value. The pressure drop for higher angles is not predictable by these means and shows a local minimum at around an angle of 135°. The reason for this minimum is unclear although it

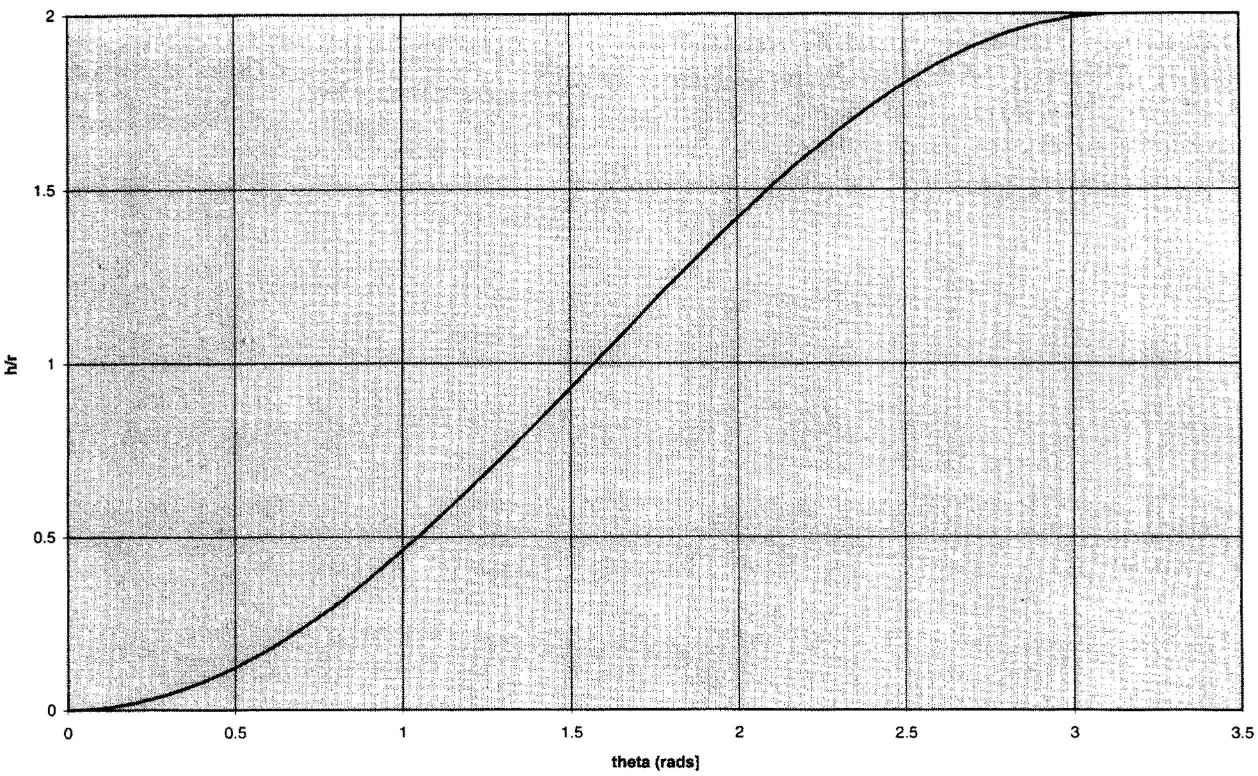

Fig. 6. h/r versus included half angle θ for the hypothesis of a planar static zone.

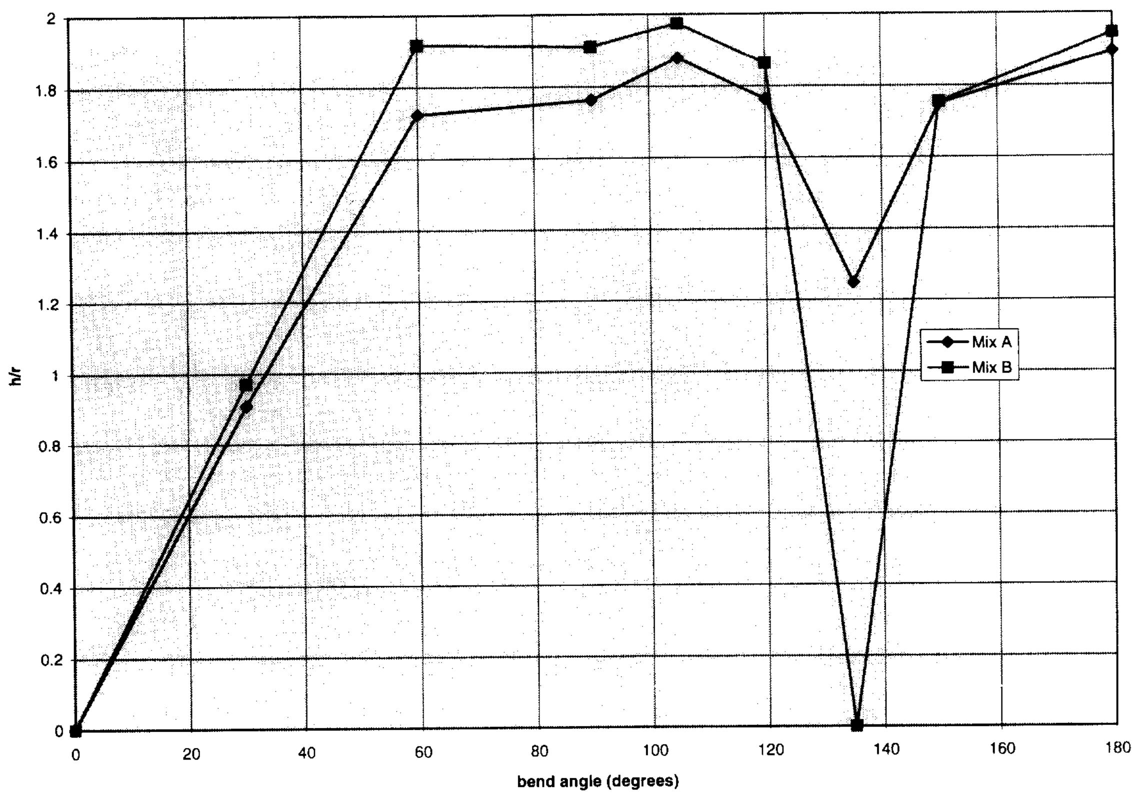

Fig. 7. Implied maximum depth h of static zone versus bend angle.

seems possible that it is connected with the dissipation of a stagnant flow region. The effect of increasing diameter is to reduce the pressure drop due to the bend to such an extent that it becomes unmeasurably small. This is in marked contrast to Hencky, which predicts that there should be no diameter effect. Further work must be carried out in order to determine the flow patterns and exact mechanisms. This may be best accomplished by means of flow visualisation. The effects of radius of curvature need also to be considered and the range of pastes tested extended before any general design guidelines can be given.

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